Correspondence

Injection Locked Phase Modulator*

This communication describes a novel type of phase modulator based on the injection locking principle.[1] The particular device described utilizes a klystron but the technique is not restricted to the microwave region of the frequency spectrum. The technique may be applied to any oscillator that has a voltage tuning characteristic. The frequency of most oscillators is somewhat dependent on the voltages applied to various parts of the oscillator circuit and heretofore many techniques have been devised to overcome this effect. The injection locked phase modulator makes use of the voltage tuning effect which may be inherent or induced by inclusion of a voltage variable capacitor, or other device.

Fig. 1 shows a one-port microwave oscillator (typically a reflex klystron) connected to a circulator and synchronized to the input signal by the injection locking technique. Repeller modulation ordinarily results in a free-running frequency deviation (FM), but with the oscillator locked to the injected signal the result is a change of phase of the oscillator output. The phase modulation is, of course, limited to less than $\pm 90°$ or synchronization will be lost; $\pm 30°$ is readily obtainable. The phase deviation is related to the system parameters by

$$\theta = \sin^{-1}\left[2Q\frac{\Delta f_0}{f_0}\left(\frac{P}{P_1}\right)^{1/2}\right] \qquad (1)$$

where

P = oscillator power
P_1 = injected power
$\Delta f_0 = f_0 - f_1$
f_0 = free-running frequency
f_1 = injected frequency
Q = figure of merit of oscillator circuit.

For fixed operating conditions.

$$\theta = \sin^{-1} K\Delta f_0, \quad \text{where } K = \frac{2Q}{f_0}\left(\frac{P}{P_1}\right)^{1/2}.$$

For a reflex klystron the free-running frequency deviation and repeller modulation are essentially linearly related for small deviations (<0.3 per cent of oscillator frequency); *i.e.*,

$$\Delta f_0 = ae_m$$

a = oscillator voltage tuning coefficient in cps/v.
e_m = modulation voltage amplitude.

Substitution gives $\theta = \sin^{-1}(Kae_m)$. Examination of the sine function shows that good linearity is obtained for $\theta < \pm 30°$. Thus, for phase deviations less than 30° there is an approximately linear relationship between

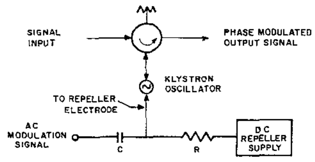

Fig. 1—Injection locked phase modulator.

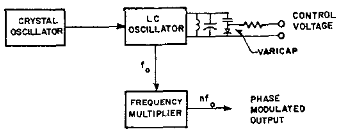

Fig. 2—Lower frequency configuration.

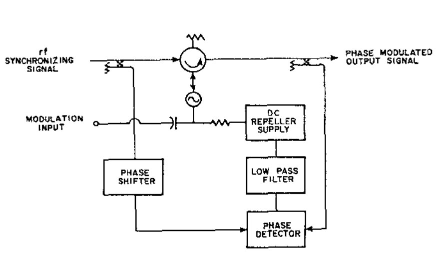

Fig. 3—Feedback to compensate for voltage and thermal drifts.

phase shift and modulation voltage amplitude. For greater deviations equalization could be made in either receiver or transmitter.

Again subject to a deviation of $\theta < \pm 30°$, the upper useful modulation frequency (corresponding to half-power frequency concept) is given approximately by

$$f_{m_{\max}} = \frac{f_0}{2Q}\left(\frac{P_1}{P}\right)^{1/2} \text{cps}.$$

For example, an X-band klystron with typical values of $f_0 = 10^{10}$, $Q = 100$, $P_1/P = 0.001$ (30-db injection ratio) would have an upper useful modulation frequency of

$$f_m = \frac{10^{10}}{2(100)(31.6)} = 1.5 \text{ Mc}.$$

Fig. 2 shows how the principle may be used at lower frequencies. A crystal controlled oscillator is used to synchronize an LC oscillator incorporating a voltage variable capacitor to enhance the voltage tuning effect. If larger phase deviations are required than are conveniently obtainable from the synchronized oscillator the frequency is multiplied by the usual techniques; the phase deviation is increased by the multiplication ratio.

Drifts in the klystron voltages, or thermal drifts may cause the modulator to drift out of synchronization. The stability of the system can be improved by placing feedback around the modulated oscillator as shown in Fig. 3. A portion of the synchronizing input signal and the modulated output signal are compared in a phase detector. The phase detector output is set to zero by the phase shifter with the modulation input disabled. With the modulation signal applied, the phase detector output will contain components from the phase modulation and from the slower voltage or thermal drifts. The low-pass filter rejects the modulation components and the drift output is fed back to the repeller power supply in the phase required to result in a correction of the drift.

Frequency Modulation

Subject to the limitation that $\theta \leq 30°$, then

$$\theta = Kae_m.$$

The instantaneous output voltage may be written

$$e_i(t) = \cos(w_c t + \theta) = \cos\phi$$

where w_c is the carrier frequency and ϕ is the instantaneous phase. The instantaneous frequency is given by

$$w_i = \frac{d\phi}{dt} = w_c + \frac{d\theta}{dt} = w_c + ka\frac{de_m}{dt}.$$

Thus if the modulation voltage is integrated before application to the locked oscillator the instantaneous frequency varies directly with e_m and frequency modulation is achieved.

$$w_i = w_c + ka\frac{d}{dt}\int e_m dt.$$

Limitation

To insure that $\theta \leq 30°$ the amplitude of the modulation voltage must not exceed some maximum value. For a sinusoidal modulation voltage, *i e.*,

$$e_m = A\cos w_m t$$

then

$$A \leq \frac{\pi/6}{Ka}.$$

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[1] R. C. Mackey, "Injection locking of klystron oscillators," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 228–235; July, 1962.

A Reflex Klystron Amplifier for Microwave Spectroscopy*

There are situations in microwave resonance studies when the microwave power level must be kept very low in order not to

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seriously disturb the systems being studied. Electron spin resonance of many systems at low temperature where saturation effects occur and cyclotron resonance of carriers at low temperatures when it is important not to heat the carrier systems appreciably, are two examples of such situations. Operation at these low power levels creates a demand for an increase in sensitivity of the detection system for the microwave spectrometer. Usually this is accomplished by employing a superheterodyne detection system in place of the more usual video detector. The complexity of such an arrangement led us to consider the possibility of using a reflex klystron to amplify the microwave bridge output signal prior to the video detector.

The reflex klystron employed as an amplifier has been studied by other investigators [1]–[6]. Stable gains of 30 db have been obtained with a voltage gain bandwidth product ($G_v \times B$) of 85 Mc in the frequency region of 10 Gc. Rather high noise figures (up to 42 db) have been attributed to shot noise in the electron beam. However, with the method of klystron mode selection reported herein the noise levels of these amplifiers may be found such that they become practical as amplifiers in low level systems.

In previous investigations with the reflex klystron employed as an amplifier it was usual to supply the klystron with potentials similar to those employed to obtain maximum output as oscillators Oscillation was then suppressed by heavy loading in the klystron output circuit. Amplification capabilities with signal output levels nearly equal to maximum power output as oscillators were obtained but the shot noise contributions were high. A review of our requirements indicated that our electron spin resonance spectrometer bridge output signals are very small compared to normal reflex klystron operating levels and that utilizing reflex klystrons with reduced potentials and corresponding low beam currents might possibly reduce the shot noise contributions. Such operation is somewhat analogous to the employment of ordinary vacuum tubes or transistors as voltage amplifiers. In some applications similar reduction of tube electron beams have been carried to the point where "starved operation" results in low level voltage amplifiers with superior noise characteristics.

According to Quate, *et al.* [4], the noise power contributed by the electron beam during its first transit through the klystron cavity is $P_{in} = i_n^2 R_s$ where R_s is the shunt resistance of the cavity and i_n is the noise current. Expressing the noise current in terms of shot noise $i_n^2 = 2\epsilon I_0 B \Gamma^2$ where Γ^2 relates to the reduction of noise current below shot noise because of multiple traverses of the beam through the klystron gap region. The thermal noise power incident to the cavity is $P_{th} = KTB$ and the noise figure becomes

$$NF = 1 + \frac{2\epsilon I_0 R_s \Gamma^2}{KT}.$$

In our first experimental arrangement a 2K25 reflex klystron having a normal beam current of 25 ma with a 300-v resonator voltage was employed. When the resonator potential was reduced to 90 v the beam current dropped to 4 ma. This should reduce the shot noise by a factor greater than six to one. Other factors including the multiple beam passages appear to further reduce the noise contribution of this amplifier.

Another contemporary development in microwave devices provided an instrument which greatly facilitates the study of microwave regenerative amplifiers. This is the backward wave oscillator which may be swept over a wide frequency range with relatively constant amplitude output One such device (Hewlett-Packard Model 686A

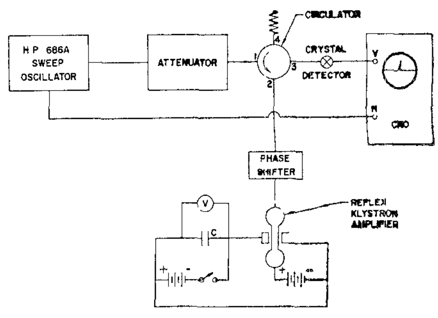


Fig. 1—Klystron amplifier test circuit.

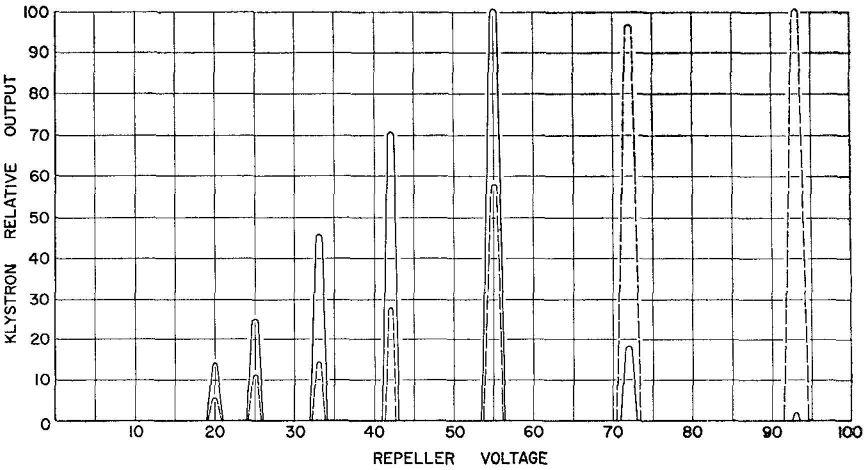


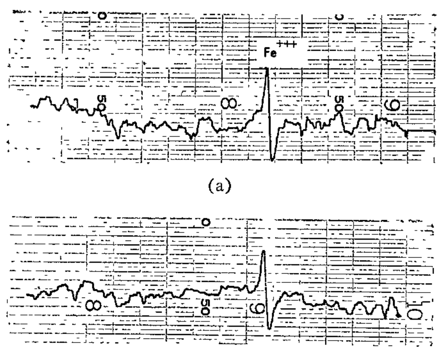
Fig. 2—Oscillation and amplification amplitudes of 723 A/B klystron.

sweep oscillator) supplied the probing signal in the amplifier test circuit configuration shown in Fig. 1. This signal generator also provides a linear sweep voltage output to deflect the cathode-ray oscilloscope spot synchronously with the microwave frequency sweep. The lower circuit shown in Fig. 1 provides a method of slowly varying the klystron repeller voltage for observation of the various operating modes. This is accomplished by shunting the repeller with a large electrolytic condenser in parallel with a high impedance voltmeter. The repeller supply voltage is applied momentarily to charge the condenser to a voltage somewhat greater than that required either for oscillation or amplification. Then as the condenser gradually discharges into the voltmeter the repeller potential drifts through the values required for the various modes. This drift is sufficiently slow so that the mode oscillation or amplification amplitudes together with their corresponding repeller potentials may be studied and tabulated. Even with the lower resonator potential, oscillation modes are observed if the loading is favorable. These are employed for reference in selecting the amplification mode as described below. Before sweeping the probing signal the klystron was loaded to favor oscillation and the oscillation amplitudes for the various modes were recorded as shown by the solid lines in Fig. 2. Then the phase control was adjusted to avoid oscillation and the swept probing signal was applied. Under these conditions the amplification characteristics for the various modes were obtained and plotted as shown by the dotted lines in Fig. 2. It will be noticed that there is a general shift toward the higher modes for the maximum amplification as compared with the maximum oscillation output modes. For this klystron (723A/B) the optimum mode for amplification would be the second next higher than the mode for maximum oscillation output, *i e.*, at a repeller potential of 93 v. At this mode the amplification is greater and it is obtained with a large reduction in beam current. The signal-to-shot noise ratio is considerably better than that obtained when a conventional selection of modes prevail.

Similar measurements were made with other 723A/B klystrons and with a Varian V-58C klystron. The shift to the higher mode region for optimum amplification was less for the V-58C as compared with the 723A/B tubes, but the trend was still evident. Also the tendency toward oscillation was greater for similar resonator potentials. From these considerations the resonator potential was reduced to $67\frac{1}{2}$ v for operating the V-58C as an amplifier. Under these conditions its gain was easily adjusted to a higher value and the ease of adjustments was superior to that of the 723A/B units.

A performance test for the reflex klystron amplifier was made by applying it to an *x*-band microwave spectrometer. The Varian V-58C tube was selected for this purpose for reasons given above. This spectrograph consists of a "magic tee" bridge with the sample cavity between the pole pieces of a large Varian electromagnet. A circulator provides the necessary isolation between input and output circuits of the amplifier. A magnet field modulation system together with a coherent detection system provides an output suitable for displaying the derivative of the absorption characteristic on a strip chart recorder.

In order to operate within a sensitivity region where predetection amplification would show advantages a crystal of "pure" sapphire was used for the sample. These crystals have unavoidable traces of iron (Fe^{3+}) and serve well for spectrographic performance checks. A typical trace obtained without the klystron amplifier showing the absorption characteristic of iron in sapphire is shown in the upper trace (a) of Fig. 3. This was obtained with about 10 mw of 9355-Mc microwave input power to the sample cavity. The second trace (b) was obtained with the klystron predetection amplifier, while employing only 10 μw input power to the sample cavity, the power incident on the crystal detector being constant.



(a)

(b)

Fig. 3—Microwave spectrographic responses. 9355 Mc, $\theta = 90°$. (a) Without predetection amplifier ($P_0 = 10^{-2}$ watts). (b) With predetection amplifier ($P_0 = 10^{-5}$ watts).

Here the second trace shows about the same signal-to-noise ratio indicating that the limiting noise of the system is not the klystron amplifier which now allows the system to operate with powers incident on the sample cavity of approximately 1/1000 of that for the straight detection. The increased sensitivity and simplicity of the klystron amplifier should make a very useful addition to simple microwave spectrometers where there is a need to operate at low microwave power levels.

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Correction to "In-Line Waveguide Calorimeter for High-Power Measurements"—Accounting for Transverse Waveguide Wall Currents*

In a recent paper[1] an analysis of calorimeter error due to standing waves in the measured section of waveguide appeared in (11)–(13) and Fig. 3. This analysis was based on the most pessimistic case of a standing-wave of waveguide dissipated power proportional to the square of the transverse H field. Engen[2] has pointed out that the resulting error expression thus obtained is applicable to a TEM wave problem; the actual waveguide calorimeter error will be smaller due to the effects of transverse waveguide wall currents. In the following an expression for the ratio of longitudinal to transverse dissipated power for the TE_{01} mode is developed and a resulting correction to the original error expression (14) is given.

The power dissipated in the walls of a rectangular waveguide is proportional to the integrated H squared fields:

$$P \propto \int_0^b (H_yH_y^* + H_zH_z^*)_{x=0}\,dy + \int_0^a (H_xH_x^* + H_zH_z^*)_{y=0}\,dx$$

where the subscript notation follows present convention. Performing this integration using the customary expressions[3] for the H components of the TE_{01} mode of propagation results in

$$P \propto (4a^3/\lambda_g^2) + (a + 2b)$$

where $4a^3/\lambda_g^2$ represents the power loss due to longitudinal wall currents and $(a+2b)$ represents the power loss due to transverse wall currents. The ratio of these two quantities can be written

$$P_{\text{long}}/P_{\text{trans}} = (\lambda_c/\lambda_g)^2(1 + 2b/a)^{-1}$$

where λ_c is the cutoff wavelength and λ_g is the waveguide wavelength.

The general effect of standing waves is covered in the original paper.[1] This effect is modified by the presence of both longitudinal and transverse dissipated powers whose undulating components are spatially out of phase by π radians. Thus if the magnitudes of the powers are equal, the power dissipation in the waveguide is uniform with longitudinal position. In any event, if there exists a standing wave of H fields in the waveguide, the amplitude of the standing wave of power dissipated in the walls F is given by the absolute value of the ratio of the difference to the sum of the longitudinal and transverse dissipated powers:

$$F = \left| \frac{P_{\text{long}} - P_{\text{trans}}}{P_{\text{long}} + P_{\text{trans}}} \right| = \left| \frac{\left(\frac{\lambda_c}{\lambda_g}\right)^2 - (1 + 2b/a)}{\left(\frac{\lambda_c}{\lambda_g}\right)^2 + (1 + 2b/a)} \right|$$

where the scaling factors involving the physical characteristics and reflection coefficient for the particular situation are omitted for the sake of clarity. The quantity F is easily evaluated, for the ratio b/a is fixed for any given waveguide geometry and the ratio λ_c/λ_g is known or easily computed for most commonly used waveguides.

Eq. (11),[1] giving the distribution of dissipated power in a waveguide as a function of longitudinal position x, the load reflection coefficient Γ and the power P_0 dissipated when $\Gamma = 0$ is now modified to read

$$P = P_0\left[1 + F\frac{2\Gamma}{1 + \Gamma^2}\cos\frac{4\pi(x - \phi)}{\lambda_g}\right]. \quad (11)$$

Thus (14),[1] expressing the maximum of the absolute value of the error of calorimeter indication due to nonzero reflection coefficient, becomes

$$|\epsilon| \le F\left(\frac{\sin L}{L}\right)^2 \frac{2|\Gamma|}{1 + |\Gamma|^2}. \quad (14)$$

It should be noted that F in the revised Eq. (14), above, is frequency dependent, so the ordinate of the error plot of Fig 3[1] is simply multiplied by a constant dependent on frequency. In S-Band waveguide, for example, this term ranges from 0.68 to 0 to 0.35 as the operating frequency is increased from the lower to the upper limit of operation of the waveguide. The revised error is now seen to be zero when $(\lambda_c/\lambda_g)^2 = (1+2b/a)$ as well as when the normalized length $L = n\pi$ corresponding to the total calorimeter length being an integral multiple of waveguide wavelengths. For a fixed length of calorimeter the error can then be zero at two or more frequencies over the operating band of the waveguide used; several error zeros may be possible for the waveguide wavelength being a submultiple of or equal to the total calorimeter length while one zero is possible when the term F goes to zero This suggests a method of broad-banding the calorimeter in that it is possible to choose a distribution of error zeros due to the above causes such that the error over any given frequency band is minimized. The calorimeter waveguide dimensions could be so chosen or its length and operating frequency so fixed that the error could be made negligible over an appreciable portion of a waveguide band.

The foregoing analysis was greatly facilitated through the suggestions of Tor Schaug-Pettersen.

Two errors appear elsewhere in the paper: the left-hand side of (13) should read "$\theta(0, \infty)$" and the caption of Fig. 5 should read "Experimental calorimeters."

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* Received November 26, 1962.

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